Application of the Complex Cepstrum to Locate Acoustic Sources Near Reflective Surfaces

C. R. Fuller,* S. Tavakkoli,† C. J. Hurst,‡ and W. F. O'Brien§
Virginia Polytechnic Institute and State University, Blacksburg, Virginia
and
K. B. Elliott¶
NASA Langley Research Center, Hampton, Virginia

The complex cepstrum is used to correct bearing estimations of acoustic sources in the presence of a reflective surface. The echo time delay, given by the complex cepstrum, then is used in conjunction with the bearing estimation and measurement array height to determine the distance to the source. Consequently, the location of the acoustic source is determined with one detection array. An automated liftering procedure is used that zeros out a block portion of the cepstrum including the echo information. The problem of the resulting distortion is alleviated by applying a coherence criterion to the recovered direct signals at each microphone. Thus, to a large degree, the operator interactive nature of cepstral processing is overcome for this application. For the test signals and geometries considered, the cepstrum is shown to accurately correct for bearing errors in acoustic signals contaminated with reflections from nearby surfaces as well as provide the necessary information to determine the source location.

Nomenclature

c	= speed of sound (343 m/s)
d_1	= source path distance
$d_2 + d_3$	= echo path distance
f	= frequency
G_{12}	= cross spectrum
h	= array height
ℓ	= microphone spacing
N	= number of independent sample sets
T	= number of frequencies considered
α	= source bearing relative to horizontal axis
Δt	= echo time delay
β	= angle of array axis to horizontal
θ	= source bearing relative to array axis
γ_{12}^2	= coherence
ω	= circular frequency

Introduction

THE complex cepstrum and the power cepstrum have been used in many situations to separate convolved signals generated by an event.[1] However, the power cepstrum is more popular in most acoustic applications. In particular in the aerospace field, the power cepstrum already has been used to correct the spectrum of sound radiated from jet engine inlets for ground reflection effects[2,3] and estimate the effects of reflections on acoustic measurements of aerospace vehicles in wind tunnels.[4] Bolton and Gold[5] have also used the power cepstrum to measure the complex surface impedance of absorptive materials typically used in the control of aircraft cabin noise.

Some proposed applications in acoustics require the recovery of the time history of the principal acoustic signal. In these situations, it is necessary to use the complex cepstrum, a process in which phase information is retained throughout.[1] For example, it is well known that the bearing of a single acoustic source in a freefield at a reasonable distance can be estimated by using the cross-spectral information of the acoustic signal measured at an array of microphones.[6] Acoustic source location experiments, however, quite often are performed in non-free-field environments where reflective surfaces may be located nearby. In this case, the bearing estimation has an induced error due to the interference of the reflected (echo) signal with the direct signal.[6] Examples are acoustic source location of aircraft in outdoor applications and the measurement of acoustic source bearings in the ocean with reflections from the ocean floor or thermal layer discontinuities.

In this investigation, the complex cepstrum is applied to acoustic signals measured in a non-free-field environment in order to remove the contaminating effect of echoes on the bearing estimation. The echo time delay, which can be determined from the cepstrum, then is used to calculate the distance to the source, thus completely determining its location. Previous work of this nature has been concerned with investigating the performance of the complex cepstrum using computer-simulated signals.[7] Results of the simulation demonstrate that the complex cepstrum still performs well for signal recovery when the reflective surface is relatively absorptive and the background noise of a moderate signal-to-noise ratio is present. The present investigation, however, is concerned with a laboratory investigation of the application of the complex cepstrum to the source location process. The cepstrum is applied to acoustic signals measured in an anechoic chamber containing a source, reflective surface, and microphone array. The bearing of the source is re-estimated once the signals have been processed, and it is shown that the cepstrum processing removes the error induced by the reflective surface for the test signals and geometries considered, as well as giving a good estimate of the source distance. In particular, a method of liftering that largely overcomes the interactive nature of the cepstrum is introduced and discussed.

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*Associate Professor, Mechanical Engineering. Member AIAA.
†Graduate Assistant, Mechanical Engineering.
‡Professor, Mechanical Engineering.
§Professor, Mechanical Engineering. Member AIAA.
¶National Research Council/NASA Research Associate.

(See Ref. 1 for an explanation of cepstral terminology.) Finally, characteristics of the cepstrum as related to signal recovery and bearing estimation are discussed.

Experimental Setup and Procedure

A schematic diagram of the experimental setup and associated test equipment is shown in Fig. 1. The tests were performed in a $2.3 \times 2.6 \times 4.0$ m anechoic chamber with a low-frequency cutoff approximately at 250 Hz to simulate a free-field environment. The acoustic signal was generated by a horn driver positioned in one corner of the chamber and measured by a microphone array consisting of two B and K ½ in. microphones positioned at the other end of the chamber. The microphones were fixed so as to have a spacing of 5.08 cm from center to center. Acoustic reflections were generated by covering a major portion of one wall of the chamber with a removable reflective surface constructed from ½ in. plywood covered with steel sheeting and braced on the rear side with 2 in. wood studs.

Test signals were generated by using a Wavetek 275 digital signal generator. The test signal used was a swept sine burst[5] sweeping from 0 to 5000 Hz in 30 ms. The required signal sequence was generated digitally on an IBM 370 mainframe computer and then down-loaded via a modem link to the memory of the Wavetek 275. It should be noted that the test signal was digitally generated at 20 kHz on the IBM 370, ensuring frequency content past 5000 Hz. On triggering the Wavetek, the test signal was recalled, converted to an analog signal, smoothed, amplified, and then fed to the horn driver. The use of the Wavetek greatly facilitated the experiments, since it enabled a wide range of test signals to be used with accurate repeatability.

Microphone signals were acquired using a Zonic 5003 two-channel signal processor. The sample rate was set at 10 kHz, giving a Nyquist frequency of 5000 Hz, and an anti-aliasing filter with a rolloff near 3000 Hz was employed. Ithaco high-pass filters set at 100 Hz were used to remove very low-frequency background noise associated with structurally transmitted noise to the anechoic chamber. A record length of 1024 points was taken, giving a sampling time of 0.1023 s. Triggering of both the Wavetek 275 and Zonic 5003 was achieved by using a square-wave generator set to allow the Zonic 5003 to acquire complete and individual records of the signal.

Once the microphone signals were sampled digitally, the data sequence was then up-loaded to the IBM 370 using the modem link. All cepstrum processing and bearing-finding calculations then were performed on the IBM computer. Control of both the Wavetek and Zonic was achieved by a Tektronix 4052 terminal and microcomputer, which could be switched to view outputs of the Zonic and IBM 370 as required. Signals were also monitored with oscilloscopes and voltmeters to ensure that no distortion was occurring in the system.

The test system ensured a great deal of flexibility in performing the experiment, combining the advantages of local control with the computing power of a mainframe computer.

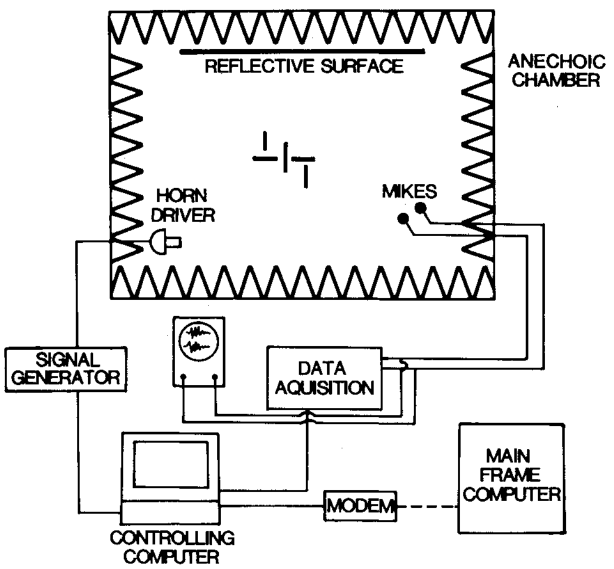

Fig. 1 Experimental apparatus.

Complex Cepstrum

Figure 2 shows the block diagram of the process used in the formation of the complex cepstrum and subsequent signal recovery. As discussed in Ref. 8, the complex cepstrum is defined as the inverse transform of the complex log of the Fourier transform of the time history. The complex cepstrum differs from the power cepstrum in that phase information is retained throughout; however, the name "complex" is somewhat misleading as the complex cepstrum is purely real in value. The final units of the cepstrum are time and are denoted "quefrency" to indicate that the result is in the cepstral domain.

The central process in the formation of the complex cepstrum is the complex log. As outlined in Ref. 8, the log process converts convolutions in the Fourier transform of signals with coherent echoes into additive components. On taking the inverse transform, the additive components are separated and the information due to the echo appears as delta functions along the quefrency axis of the complex cepstrum.[1,8] The information due to the direct signal is similar in characteristic to an autocorrelation function that is centered at a quefrency of $\tau = 0$ in the complex cepstrum.

Thus, since the information due to the echo is compressed into delta functions, it is possible to "lifter" these delta functions by zeroing them out of the cepstrum along the quefrency axis. The cepstrum process then is reversed, and the final result is the recovery of the wavelet in the time domain without the presence of its echo. Different types of liftering can be used.[1] For this investigation, a block lifter was used in which a complete quefrency block of the cepstrum including all the delta functions is set to zero. The advantage of this method is that it precludes the rather operator interactive nature of the liftering process in first identifying the delta function locations and then deciding whether to remove them by zeroing, interpolation, etc. The width of the block is simply chosen as a constant and calculated to be wide enough so that it will include all delta functions for a minimum echo delay time. Obviously, this type of liftering removes information associated with the direct signal and results in some distortion of the recovered wavelet. However, as discussed later, this distortion problem can be circumvented in bearing estimations by using a coherence test, and the process enables a relatively automated procedure to be used. Similarly, the more broadband/random content in the signal, the more cepstral information will be concentrated at low frequencies, and less distortion will occur from block liftering. Thus, the extent of distortion depends on signal characteristics and for many

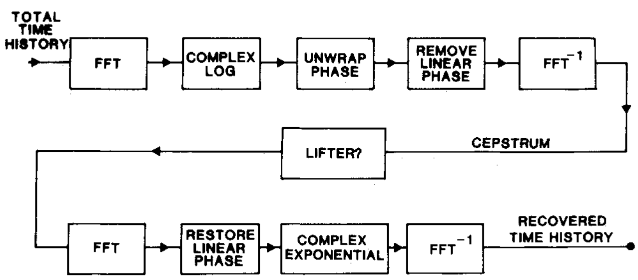

Fig. 2 Cepstal recovery process.

applications could be negligible. For example, Refs. 4 and 9 demonstrate that aerospace signals consisting of pure tones on broadband noise will have a power cepstrum with most of the signal information compressed in the low quefrencies.

Other aspects of the complex cepstrum that are important are shown in Fig. 2. In particular, the complex logarithmn is a multivalued function leading to discontinuities in the phase curve. These discontinuities must be removed by unwrapping the phase, and this was achieved here by using an algorithm developed by Tribolet.[10] Similarly, the complex function must be Hermitian symmetric, and this necessitates removing a linear component from the phase at the midpoint to pull it down to zero. On reversing the cepstrum process, the linear component is reinstalled to ensure accurate prediction of overall time delays in recovered wavelets.

The performance of the complex cepstrum on signal recovery already has been studied using computer simulation.[7] In general, as mentioned before, the main requirement is that the test signal have a reasonable bandwidth so that the direct signal appears at low quefrency in the cepstrum. However, the cepstrum combined with block liftering was found to give quite accurate recovery of wavelets in the presence of distortion and noise, provided these are not too excessive.[7] Other processing aspects of the cepstrum can be found in Refs. 1 and 5.

Bearing Estimation Algorithm

The calculations of the bearing estimation were performed on the mainframe IBM 370 machine. A block diagram of the processes is given in Fig. 3. The top of the diagram starts with data acquisition of the signal at each microphone by the Zonic signal processor. In the more general case, N data sets of each signal are sampled and transferred to the IBM. The next three steps in the algorithm implement the recovery of the direct signal by forming a complex cepstrum, liftering (if required), and reversing the process. For the experiments reported, five data sets were obtained and the liftering was standardized to zeroing the cepstrum from quefrencies of 3.9–98.3 ms. This limited this application to echo time delays of greater than 3.9 ms.

Once the direct signals are recovered, their autospectrums and cross spectrums are estimated and averaged over the N samples to remove noise and form a reasonable statistical sample. As outlined in the Appendix, the bearing angle of the source then is calculated from the microphone array cross

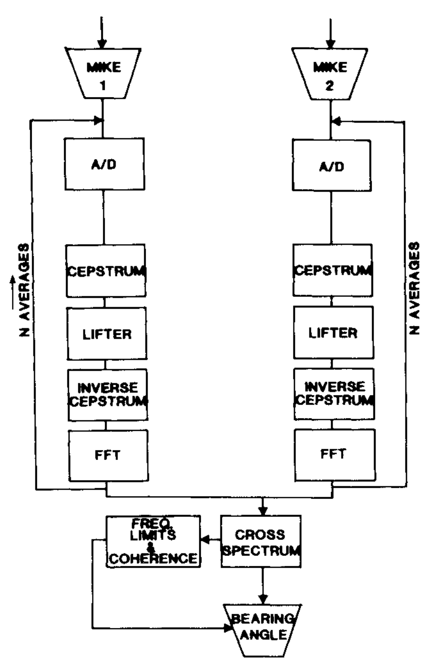

Fig. 3 Bearing-angle calculation procedure.

spectrum. In theory, any one frequency of the cross spectrum can be used to calculate the bearing angle. However, the practice assumed here is to obtain a bearing angle from the average of all the bearings calculated at each frequency, or bearing angle

$$\theta = \sum_{i=1}^{T} \theta_i / T$$

where i is the frequency index number and T the total number of frequencies considered. This procedure tends to average out errors in the bearing estimation calculations.

Before the bearing-averaging procedure is implemented, it is necessary to satisfy several criteria on the accuracy of the cross-spectrum information. Upper and lower cutoff frequencies were chosen based on microphone spatial aliasing and the accuracy of measurement of the phase difference between the microphones, respectively, as

$$c/4\ell < f < c/2\ell \tag{1}$$

where ℓ is the microphone spacing and c the speed of sound. Thus, for a microphone spacing of 5.08 cm, the upper and lower frequencies are 1688 and 3375 Hz, respectively. Note that the aliasing filter used prior to data acquisition must be set at a cutoff frequency to prohibit both spatial and frequency aliasing.

Thus, all cross-spectral information outside these limits was discarded during the averaging process. As mentioned previously, the block liftering used tends to distort the recovered signal at each microphone. Thus, in order to overcome this problem, the coherence between the recovered microphones signals was calculated, and all frequencies for which the coherence γ_{12}^2 was less than 0.98 were discarded. The remaining frequencies were then averaged and the resulting bearing estimation calculated. Thus, the third criterion is

$$\gamma_{12}^2 > 0.98 \tag{2}$$

Source Distance Estimation

Figure 4 gives the geometry of the test arrangement, limited to one plane.

If the echo delay time, estimated from the complex cepstrum by the location of the first delta function[1] (rahmonic), is Δt, then

$$(d_2 + d_3) - d_1 = c\Delta t \tag{3}$$

where c is the speed of sound.

By examining the geometry of the test arrangement and its image in the reflecting plane, it can be seen that

$$[2h + d_1 \sin(\alpha)]^2 + [d_1 \cos(\alpha)]^2 = (d_2 + d_3)^2 \tag{4}$$

Solving for $(d_2 + d_3)$ from Eq. (3) and substituting in Eq. (4) gives the source distance as

$$d_1 = [(c\Delta t)^2 - 4h^2]/[4h \sin(\alpha) - 2c\Delta t] \tag{5}$$

where h is the detection array height. As shown in Fig. 4, if the detection array axis has an orientation of β relative to the horizontal, then $\alpha = \theta - \beta$, where θ is the source bearing measured relative to the array axis.[6]

In essence, it can be seen from Fig. 4 that the image of the detection array in the reflective surface effectively provides another source-bearing estimation from a different location. Thus, the source distance can be obtained by triangulation. This implies that to obtain a reasonable source distance estimation the array must be located well above the reflective surface.

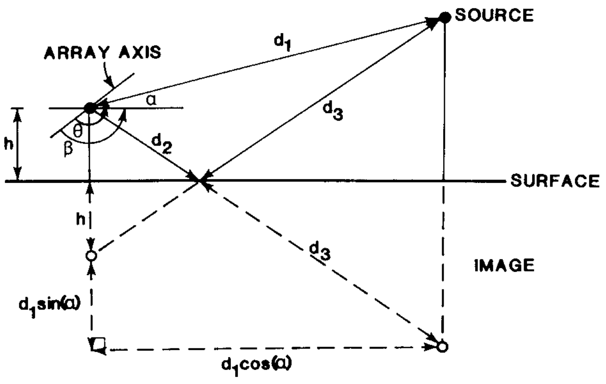

Fig. 4 Test geometry.

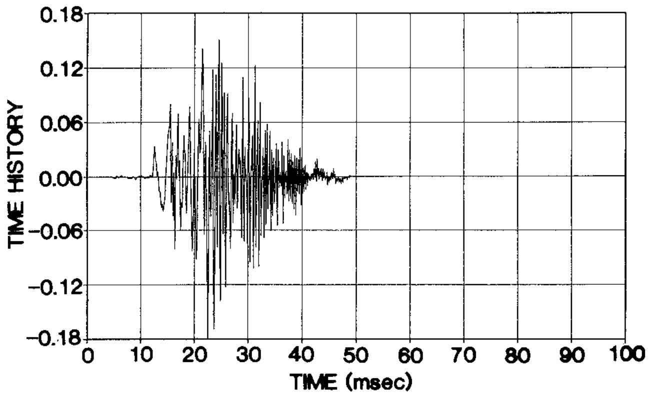

Fig. 5 Time history at microphone 1.

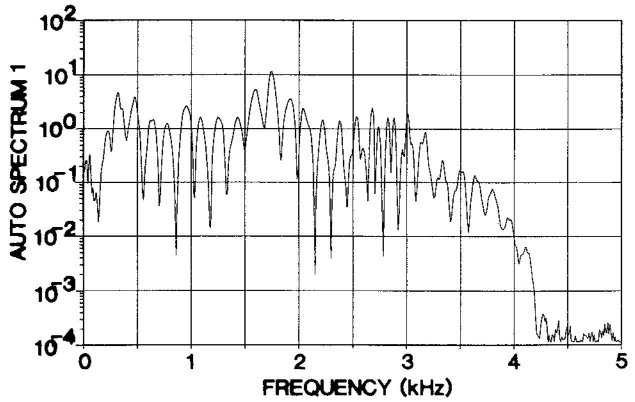

Fig. 6 Autospectrum at microphone 1.

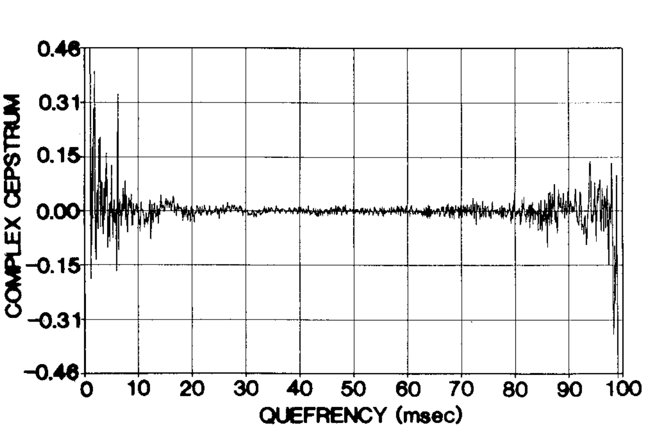

Fig. 7 Complex cepstrum at microphone 1.

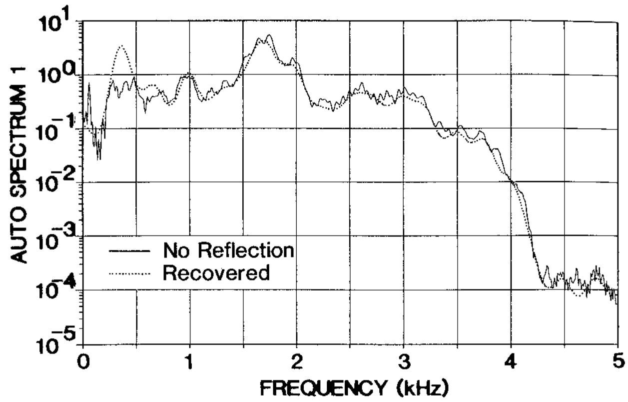

Fig. 8 Recovered autospectrum at microphone 1.

Experimental Results

For the experimental results presented here, the source was positioned at an angle of $\theta = 146$ deg to the microphone array (corresponding to the notation of Ragone[6] discussed in the Appendix). As the array axis was positioned at an angle of $\beta = 148.7$ deg to the horizontal, this gave a theoretical source-bearing angle relative to the horizontal of $\alpha = -2.7$ deg. Other relative dimensions were that the array height was 2.39 m, whereas the distance to the source d_1 was 3.80 m. The length ratio between the direct and reflected acoustic paths was 0.637, thus giving a theoretical echo time delay of 6.3 ms.

Figure 5 shows the time history of the acoustic signal measured at microphone 1 with the reflective surface present in the chamber. Although the swept sine function from the Wavetek was flat with frequency, the amplitude of the test signal varied due to the frequency response of the speaker. It is also impossible to distinguish the direct signal and echo as they are overlapping in time. The second half of the time history has been appended with zeros in order to alleviate cepstral aliasing.[1,5]

Figure 6 presents the autospectrum of the time history of the previous figure. The main characteristic of this figure is the periodic ripple associated with successive destructive and constructive interferences of the echo with the direct signal. Overall, the spectrum can be seen to be relatively flat up to approximately 3000 Hz at which point the anti-aliasing filter takes effect.

Figure 7 gives the corresponding complex cepstrum. For this graph, the first few points of the cepstrum are not plotted in order to expand its range and facilitate viewing of its characteristics. A delta function can be seen in the cepstrum located at a quefrency of 6.25 ms. This implies that the echo delay time is also 6.25 ms to be compared with the theoretical value of 6.3 ms. The second rahmonic (delta function) can also be distinguished at a quefrency of 12.5 ms and so on.

The cepstrum of Fig. 7 was block liftered in the liftering algorithm from quefrencies of 3.9–98.3 ms, the cepstral process was reversed, and the direct signal time history recovered. The mean-square error[11] of the recovered signal was calculated by comparing the microphone time history without the reflective wall installed. A value of 0.013, normalized to the maximum amplitude squared of the time history, was obtained, indicating reasonable recovery of the direct wavelet. However, some distortion was evident. The autospectrum of the recovered wavelet then was calculated and is plotted in Fig. 8. Also plotted in the figure is the measured autospectrum when the reflective surface is not present, and thus no liftering was carried out on the cepstrum. The recovered autospectrum can be seen to be a very good estimate of the direct spectrum over most of the frequency range. The main effect of the block liftering is a smoothing of the autospectrum.

Figure 9 presents the coherence calculated between the recovered signals at each microphone. Without liftering, the coherence is close to unity across most of the frequency range. Block liftering can be seen from Fig. 9 to cause "drop outs" in the coherence due to signal distortion, and it is at these frequencies that the cross-spectral information is discarded in the bearing calculation.

The bearing of the source was calculated using the procedure outlined previously. Figure 10 gives a bearing plot without the reflective surface in place and thus no liftering carried out. The polar plot is constructed so that lines of equal length are drawn at the angle computed for each of the frequencies that satisfy the previous three criteria of Eqs. (1) and (2). The average bearing angle for this situation was calculated to be 144.6 deg relative to the array axis. The distribution of the individual bearings are reasonably compressed around the mean value. The 95% confidence limits for the distribution were calculated to be ± 0.489 deg indicating a good degree of accuracy of detecting the correct angle of the source.

The next test was to install the reflective surface in the chamber and remeasure the source bearing. In this test, liftering was not performed to remove the effect of the echo. Figure 11 shows the bearing plot and the (average) estimated bearing under these conditions. It is apparent that the presence of the reflective surface has skewed the bearing estimate by approximately 24 deg due to the resultant echo interfering with direct signal. Likewise, there is a larger scatter associated with the individual bearing rays, and the 95% confidence limits are consequently ± 5.46 deg. This result was thought to be due to a signal-to-noise ratio problem at frequencies where the echo interferes destructively with the direct signal or nonlinearities associated with the finite width of the reflecting surface.

The data shown in Fig. 11 were then cepstral processed and liftered to remove the echo information. Figure 12 shows the bearing estimation with the echo removed. The source bearing in this case is estimated to be 144.53 deg with 95% confidence limits of ± 1.57 deg, again indicating a high degree of confidence in the result. Thus, the cepstral process can be seen to successfully remove the contaminating effect of the echo and correct the bearing estimation. Likewise, the number of bearings with large scatter has been reduced as these are associated with the destructive interference of the echo on the direct wavelet and thus have been removed by liftering. It is likely that a further improvement in the accuracy of the bearing estimation could be made by increasing the number of elements in the array.

Finally, the source distance was calculated from the estimated bearing angle, the measured echo delay time, and detection array height using Eq. (5). The corrected measured angle $\alpha = 144.5 - 148.7 = -4.2$ deg.

Calculating d_1 from Eq. (5) gives the estimated source distance as 3.66 m as compared to the actual value of 3.90 m.

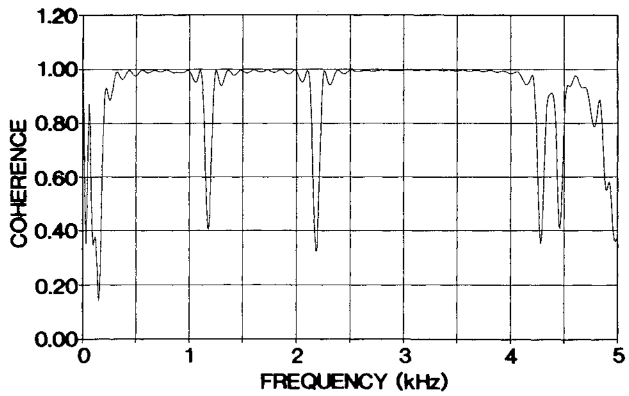

Fig. 9 Coherence of recovered signals at microphones.

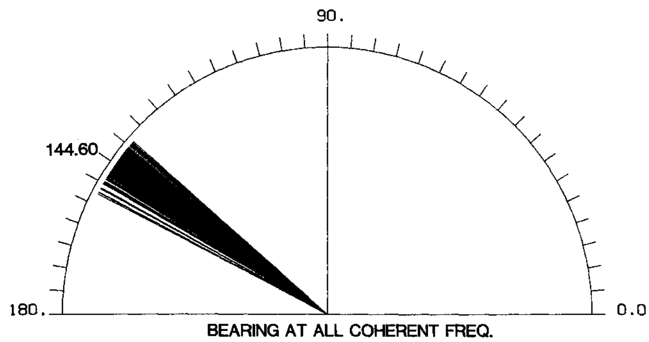

Fig. 10 Bearing plot, no reflective surface.

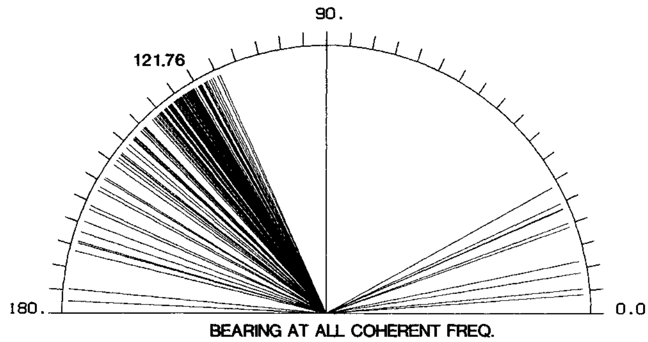

Fig. 11 Bearing plot with reflection, no cepstral processing.

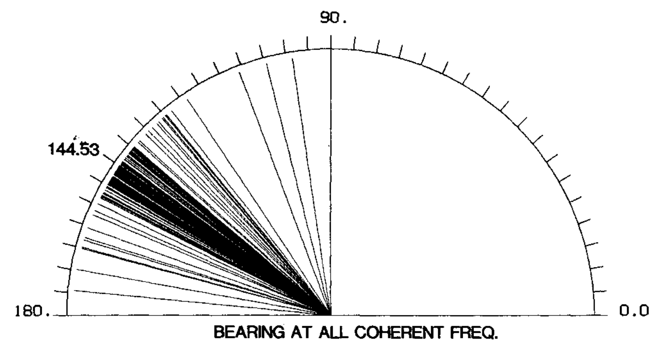

Fig. 12 Bearing plot with reflection and cepstral processing.

Thus, the source distance was measured with a 6% error. Obviously this could be improved if a more accurate bearing estimation was made.

Conclusions

An experiment has been performed to investigate the application of the complex cepstrum to the location of acoustic sources in the presence of a reflective surface. The results demonstrate that, for the test geometries and signals considered here, the complex cepstrum can be used successfully to remove the contaminating effects of an echo on bearing estimation. Likewise, it has been demonstrated that the echo delay time, which also can be obtained from the cepstrum, can be used to determine the source distance, thus completely specifying the source location in one plane with a one-point array. For this particular application, the cepstrum process has been made relatively automatic by using a block liftering technique. The resultant distortion problems in the recovered wavelet introduced by block liftering are overcome by using a coherence criterion to indicate which frequencies of the microphone cross spectrum retain accurate information. In effect, the block liftering was found to smooth the recovered autospectrum.

Acknowledgments

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Appendix: Bearing Estimations from Cross-Spectral Information

The application of the cross spectrum of a two-microphone array to the bearing estimation of an acoustic source in an anechoic environment has been studied in detail by Ragone.[6] In particular, the bearing angle θ of a source at a reasonable distance from the array is given by

$$\cos(\theta) = \frac{c}{\omega \ell} \tan^{-1} \frac{Im[G_{12}]}{Re[G_{12}]} \tag{A1}$$

where c, ω, and ℓ are the speed of sound, frequency, and microphone spacing, respectively. The averaged cross spectrum of the signals acquired at microphones 1 and 2 is G_{12}. In Eq. (A1), θ is measured relative to the axis of the microphone array, from microphone 2 toward microphone 1.

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